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ABSTRACT

The purpose of this work is to investigate, to develop, to formulate e to compare possible navigation solutions through the using of measurements obtained by GPS receivers in space environment. There are different methods to obtain navigation solution by GPS: algebric methods, geometric methods and statistical methods. This paper shows the development of the methods and the equations.

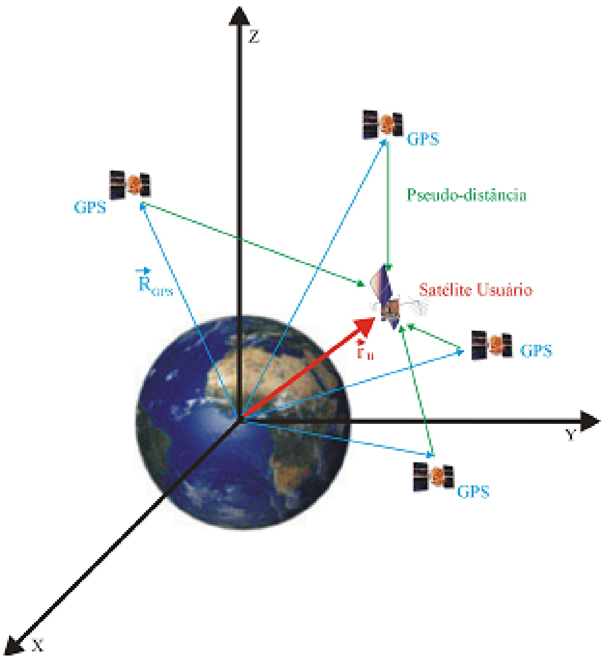
INTRODUCTION

To determinate an artificial satellite orbit means to determinate the the satellite position and velocity in relation to an inertial reference system, using a set of measurements. These observations can be obtained through tracking systems on ground or sensors on board of the space vehicle. The GPS is a satellite navigation system and its main purpose is to determinate position, velocity and the time with high precision. Its main purposes are to aid in radionavigation in three dimensions with high precision in position, navigation in real time. It also has global coverage and fast obtainment of informations sent by the satellites.

NAVIGATION SOLUTION (POSITION) BY GPS

The determination of the position (navigation solution) is done through the triangulation method (see Fig.1). The GPS receiver on board of the user satellite receives signals which allow the calculation of the position of the GPS satellites in relation to a system of reference, and measures the signal transmission time, that allows the calculation of the distance between the receiver and the GPS satellite that sent the signal. With this, the receiver is able to calculate its position in relation to a system of reference.

Figura 1: Ilustration of Triangulation Method.



GPS OBSERVATIONS MEASUREMENTS

The GPS observables are the distances obtained from the measured differences of time or phase based on comparison between the received signals and the signals generated by the receiver. The observables furnished by the GPS are pseudorange and the carrier phase.

Pseudorange is the difference of the reception time and the transmition time of a satellite signal. It is given by:

$$\rho_c = \rho + c\left[\Delta t_{GPS}(t) - \Delta t_u(t)\right] + \Delta_{ION} + \Delta_{TRO} + \varepsilon = c\tau \qquad (1)$$

where $\rho = \sqrt{(x_{GPS} - x)^2 + (y_{GPS} - y)^2 + (z_{GPS} - z)^2}$.

Carrier phase is defined as the difference between the carrier phase of the GPS satellite received by the receiver antenna and the phase of the receiver internal oscillator on the measured epoch. It is given by:

$$\phi_c = \rho + c[\Delta t_{GPS}(t) - \Delta t_u(t)] - \Delta_{ION} + \Delta_{TRO} + \lambda N + \varepsilon \qquad (2)$$

METHODS TO CALCULATE THE NAVIGATION SOLUTION

Geometric method: they are basically simple methods that give a rustic initial estimative of the navigation solution, that can be refined later through other methods or through statistician filters (Lopes and Kuga, 1997).

An approximated linear solution can be found as follow:

$$y_{pi}^2 \approx |r - R_i|^2 = r^T r + R_i^T R_i - 2R_i^T r \qquad (3)$$

where r is the position vector of the user satellite, R_i is the position vector of the I-th GPS satellite.

$$y_{pi}^2 - < y_{pi}^2 > \approx R_i^T R_i - < R_i^T R_i > -2[R_i - < R_i >]^T r \qquad (4)$$

Defining:

$$\widetilde{R} \equiv [\ldots : R_i - < R_i > : \ldots]^T \qquad (5)$$

and

$$z \equiv \frac{1}{2}\left[\ldots : R_i^T R_i - < R_i^T R_i > -\left(y_{pi}^2 - < y_{pi}^2 >\right) : \ldots\right]^T \qquad (6)$$

it results in:

$$\tilde{R}r \approx z \, . \tag{7}$$

Algebric method: the level of precision of the navigation solution based on GPS mesuarements depends on the type of the collected mesuarement, the duration that the measurements were collected and how they were modeled and processed.

If the GPS measurements are processed in real time, the most simple positioning problem is to solve simultaneously a set of navigation equations based on one frequency.

One of the algebric methods to obtain the navigation solution is Bancroft's (1985). This method presents capacity to include *bias*, bigger algebric complexity and it is efficient for computer usage and numerically stable.

The equations of the GPS navigation system are usually solved with the application of Newton's method:

$$x_{n+1} = x_n + H^{-1}(t - f(x_n)) \tag{8}$$

where x is a vector taking the user position coordinate together with the clock correction, t is a measurement vector of four pseudorange and H is a partial derivative matrix, $H = f_{x}.$.

Let's do x and $\{s_i : 1 \le i \le n\}$ indicate the position coordinate of the satellites and the user in the cartesian coordinate system; and $\{t_i : 1 \le i \le n\}$ the measurements of pseudorange collected by the user of each the n satellites:

$$t_i = d(x, s_i) + b \tag{9}$$

where $d(x,y)$ is the distance from x to y and b is the clock correction. We define the vectors from datas of 1 x 4 rows:

$$a_i = (s_i^T t_i)^T \, , 1 \le i \le n \, . \tag{10}$$

Defining:

$$A = (a_1 \quad a_2 \quad a_3 \quad \ldots \quad a_n)^T \, , \tag{11}$$

$$i_0 = (1 \quad 1 \quad 1 \quad \ldots \quad 1)^T , \qquad (12)$$

$$r = (r_1 \quad r_2 \quad r_3 \quad \ldots \quad r_n)^T , \qquad (13)$$

where $r_i 1 \le i \le n$ is calculated by $r_i = \langle a_i , a_i \rangle / 2$.

We calculate the generalized inverse as $B = (A^T WA)^{-1} A^T W$, where W is the positive symmetrical matrix. Then we calculate the vectors 1 x 4 *u* and *v* from:

$$u = Bi_0 \text{ and } v = Br , \qquad (14)$$

altogether the coefficients *E, F, G* are defined by:

$$E = \langle u,u \rangle , \; F = \langle u,v \rangle - 1 , \; G = \langle v,v \rangle . \qquad (15)$$

Solving the squared equation:

$$E\lambda^2 + 2F\lambda + G = 0 \qquad (16)$$

for the pair of roots $\lambda_{1,2}$:

$$y_{1,2} = \lambda_{1,2} u + v \qquad (17)$$

Then, with the identification

$$y^T = (x^T - b)^T \qquad (18)$$

also the pair x_1,b_1 or the pair x_2,b_2 will solve the GPS problem to the user position and the clock correction. To difference the real solution, we replace back the equations defining the original pseudoranges.

Statistical method: this method uses measurements redundance to statistically obtain the best solution. The signals can be suitably received and decoded by the GPS receivers. If the signals are suitably received, a set of three satellites would be enough to supply the geometric difficulties.

However, mainly due to the drift clock a bias is introduced in the computed distance (pseudorange), and then it is essential the using of four satellites.

The ORBEST method (Lopes and Kuga, 1988), consists in the resolution otimization problem of the following:

To minimize $L^*(\boldsymbol{r}, \rho_i, \Delta y) = L(\boldsymbol{r}, \rho_i) + \frac{1}{2} a^* \Delta^2 y,$ (19)

Subject to $\rho_i^t \rho_i = (y_{pi} + \Delta y)^2, \quad i = 1, 2, \ldots, n,$ (20)

Considering that: $\{a^*, (R_i, y_i, a_i), i = 1, 2, \ldots, n; \quad n \geq 4\}$ (21)

Where $\boldsymbol{r}$ is the position vector of the user satellite (target); R_i is the position vector of the I-th GPS satellite; ρ_I is the relative position vector relative of the user satellite in relation to the I-th GPS satellite; y_i is the pseudorange measured from the I-th GPS satellite; and a_i is a positive weight.

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